

THE POST-DETERMINED BLOCK UNIVERSE

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ABSTRACT. A series of reasons to take quantum unitary evolution seriously and explain the projection of the state vector as unitary and not discontinuous are presented, including some from General Relativity. This justifies a reevaluation of the currently accepted limits of unitary evolution and of relativistic spacetime. I argue that unitary evolution is consistent with both quantum measurements and the apparent classicality at the macroscopic level. This allows us to take the wavefunction as ontic (but holistic), but a global consistency condition has to be introduced to ensure this compatibility. As a consequence, Quantum Theory turns out to be consistent with a definite general relativistic spacetime, eliminating some of the tensions between the two, usually considered as reasons to quantize gravity. But the block universe subject to global consistency gains a new flavor, which for an observer experiencing the flow of time appears as “superdeterministic” or “retrocausal”, although this does not manifest itself in observations in a way which would allow the violation of causality. However, the block universe view offers another interpretation of this behavior, which removes the tension with causality.

Such a block universe subject to global consistency appears thus as being post-determined. Here “post-determined” means that for an observer the block universe appears as not being completely determined from the beginning, but each new quantum observation eliminates some of the possible block universe solutions consistent with the previous observations. I compare the post-determined block universe with other views: the evolving present view, the block universe, the splitting block universe, and the growing block universe, and explain how it combines their major advantages in a qualitatively different picture.

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1. INTRODUCTION

The most successful theories we have, Quantum Mechanics (QM) and General Relativity (GR), seem to be mutually inconsistent, to have some internal tensions, and also some tensions with other observed phenomena. Should we, in order to solve these tensions, make radical changes, or should we rather take a more conservative route? If we learned something from the history of physics, it is that even the best theories we have tend to be replaced by better theories, so we should expect that this will likely continue to happen. However, at least some important principles survived when better theories replaced the old ones. To get results quicker, it may be tempting to focus on solving a particular problem, for example the measurement problem in QM, and sacrifice in the process more general well-tested principles, for example unitary evolution or relativistic invariance. But before trying to solve such problems, it may be useful to see how far we can push the validity of each principle, to see where they really break down, rather than declaring them dead the first time we meet an obstacle. In general, theories trying to solve the internal tensions of QM are what we call *interpretations of QM*. With the exception of the Many Worlds Interpretation and the proposal discussed in this article, the major interpretations propose new physics – either extensions with “hidden variables”, or modifications of the unitary dynamics described by the Schrödinger equation.

In this article, I apply this conservative route to unitary evolution, to push it as far as possible and see if it really breaks down during measurements, if it is really necessary to invoke a wavefunction collapse. This is a natural continuation of some results developed in previous works of the author [93, 100, 94, 102, 103], and of some earlier ones developed by Schulman [86, 87, 88, 89] (I will explain the difference between these two approaches in section §3). At the same time, I try to be conservative about GR as well. In the case of GR, the most common proposals are to modify the theory, or to obtain it as a limit of a supposedly better theory, most likely a theory of Quantum Gravity. But, again, before doing this, it would be useful to push the limits of the principles of GR. There are too many possibilities to take into account when advancing towards Quantum Gravity – even if most of those that we know are incomplete or have problems – and the only firm ground we have are the principles that we know to be well tested, so it may help to find out where their limits are, and see if they can agree with each other.

There are strong reasons to expect that the wavefunction is real, ontic, rather than a mere collection of probabilities about unseen states or variables or outcomes [90, 55, 29, 30, 79, 54, 81, 74], and we will see some arguments supporting this in section §2. In section §3, I will present some problems which would arise if there is a wavefunction collapse taking place like a discontinuous projection. The possibility that what appears to be collapse takes place by unitary evolution alone is discussed in section §4. This turns out to be possible, but it can only happen for a small subset of the Hilbert space. This limitation of the allowed solutions gives the appearance of a conspiracy among the initial conditions of the quantum particles, or of retrocausality, depending on how you describe the order of the events in time. Retrocausal interpretations have been known for a long time to be able to save locality [35, 49, 80, 31, 32, 67, 118, 77, 106, 6, 78, 107, 1, 2, 4, 27, 28, 119]. Bell’s theorem forces us to choose between nonlocality, which is at least ontologically at odds with Special Relativity (SR) even if satisfies the no-signaling principle, and retrocausality or other ways to violate statistical independence, which are or can be consistent with relativistic local causality, do not signal backwards in time, but require the past to depend

on the future as well as the other way around. Can there be a principle which, rather than modifying the dynamics or extending the theory with new variables, just keeps the “good” solutions of the Schrödinger equation, and exhibits this apparent retrocausality in a natural way? In section §5 I argue that this can be achieved by a global consistency condition, without requiring modifications of the important principles, and in fact it can save them. Not only is this not in tension with Special or General Relativity, but in fact the latter can provide an explanation, or at least a framework, for the former. Section §6 discusses how probabilities can arise, but unfortunately a derivation of the Born rule in this framework is still an open problem. Some apparent tensions between QM and GR are discussed in section §7, and it is shown that in this framework some of them are naturally resolved or avoided.

Along this discussion, a version of the block universe emerges as the natural framework for this interpretation of QM (section §8). The observer experiencing the flow of time will have an alternative to the retrocausal description, in which the state of the universe is initially undetermined, and becomes more and more determined with each new measurement. From the bird’s eye view of an observer outside of time, this appears merely as the result of the global consistency condition. I call this framework the *post-determined block universe* because of the necessity to take into account all of the constraints imposed by quantum measurements, before determining the global solution.

2. IS THERE A REALITY BEYOND THE WAVEFUNCTION?

Schrödinger’s equation was discovered when trying to explain the energy levels and structure of the atoms [83]. It was then extended to the relativistic case, including creation and annihilation of particles. The resulting Quantum Field Theory (QFT) provides an accurate description of the behavior of particles. If this would be all there is to be explained, Quantum Theory would be the perfect theory. A simple story that explains almost everything: many-particle states, which evolve in time by being transformed by a unitary operator. But this is not the full story.

In the following I will refer to the evolution equation by the name of Schrödinger, even though the relativistic versions are due to Klein-Gordon, Dirac, and others, and even if field quantization is in place. I will do this for simplicity, based on the fact that these equations have the general form of a Schrödinger equation or the square of such an equation, and the evolution is unitary, of the form (1).

$$(1) \quad |\psi(t)\rangle = \hat{U}(t, t_0)|\psi(t_0)\rangle,$$

where $|\psi(t)\rangle$ is an operator defined on a suitable Hilbert space $\mathcal{H}$, and $\hat{U}$ is the unitary evolution operator. In QFT the state vectors $|\psi(t)\rangle$ are replaced by linear operators $\hat{\psi}$ acting on a special vacuum state $|0\rangle$ to create the states $|\psi(t)\rangle$, and their evolution is still unitary.

But there are some important problems with Quantum Theory which we have to resolve. On the one hand, gravity, particularly as it is understood in Einstein’s Theory of General Relativity, seems to not fit in this description. On the other hand, the world appears macroscopically classical, and measurements have definite outcomes. A quantum observation always finds the observed system to be in an eigenstate of the Hermitian operator corresponding to the observed property, and the outcome to be an eigenvalue. Since

the probability that the state was already an eigenstate by chance is zero, there seems to be an inconsistency between the unitary evolution and the fact that the observed system is always found to be in an eigenstate of the Hermitian operator. To solve this inconsistency, it is proposed that the measurement itself is accompanied by some projection of the state vector which does not seem to follow from the Schrödinger equation itself, and even breaks it. This solution allows us to obtain a prediction of the probabilities associated to each outcome of the measurement, but we still need to understand how or why it happens, so we have what is called “the measurement problem”. These problems are indicators that there’s more to the story than it seems.

Bohr’s solution was to take as given the classicality of the macroscopic level [20], and he described quantum measurements by assuming the apparatus and the outcomes of the measurements to be classical, and by accepting the wave-particle duality. Heisenberg’s views cross-pollinated his ideas [63], leading to the Copenhagen Interpretation, which avoids discussing anything but the outcomes of the measurements. It is often considered that restraining the discussion to the outcomes of the measurements makes the problems vanish, or even that these are not real problems. But not everyone is convinced that this makes the problems go away, as we know from the objections by some, including Einstein and de Broglie. Also Schrödinger explained the problem of classicality in his famous cat thought experiment [84].

Avoiding to discuss a problem is useful in solving other independent problems, but it does not make it go away. These foundational problems are important in particular if we really want to understand our world, particularly how General Relativity and Quantum Theory can coexist in a consistent way.

The fact that they are indeed problems becomes apparent when we try to understand what the wavefunction is. When we are talking about atoms or other systems of particles which are stationary or even interact, the wavefunction seems to be like a classical field if the state is separable, or at least like a classical field on the configuration space in general. Something has to be real there, something has to carry the energy and momentum, and curve spacetime according to the Einstein equation. The classical limit of quantum electrodynamics comes with the appropriate stress-energy tensor, which seems to be distributed in space like the wavefunction is. The inertia properties of matter, even when they are not subject to quantum measurement, are consistent again with such a stress-energy tensor.

But when we perform a measurement, the wavefunction seems to turn into a probabilistic device, whose role is to predict the probabilities to jump from one state to another during a measurement. The probability is given by the squared scalar product between the state before and the eigenstate after the measurement (which is in fact the *Born rule* [21] extended from positions to any observable).

Is this a simple ambiguity, or a contradiction? How can we interpret the wavefunction as being ontic when we do not look at it, and epistemic during measurements? Obviously the Copenhagen Interpretation chooses a quick way out, by denying the reality of the wavefunction (or at best claiming that it is irrelevant), to avoid this contradiction.

This apparent contradiction is visible in the more rigorous mathematical formulations of Quantum Mechanics by Dirac [39] and von Neumann [113]. Accordingly, the state vector is always well defined and has a unitary time evolution, according to the Schrödinger

equation, but when it is measured, it projects to an eigenstate of the operator corresponding to the observable, according to the Born rule. The apparent conflict becomes manifest when we try to formulate the theory in a mathematically rigorous way, and the infamous *wavefunction collapse* seems to be unavoidable.

As the father of General Relativity, Einstein realized this tension between realism and the purely epistemic view of the Copenhagen Interpretation in its full depth. He never ceased to hope that there is a better explanation, and considered that Quantum Mechanics is incomplete in some sense. He made his most concrete formulation of the problem together with his collaborators Podolsky and Rosen in [41], where they proposed the famous *EPR experiment*.

There are attempts to regard particles as well-localized to a point, and to interpret the wavefunction as just giving the probability to find the point-particle at given positions. But even the most successful theory that postulates point-particles, *de Broglie-Bohm theory* [37, 36, 17, 40], attributes an ontological status to the wavefunction. Regardless of the proposed interpretation of particles as points, to avoid inconsistencies with the experiments, we have to assign physical properties like charge and mass densities, and ultimately all physical properties except for definite positions, to the wave itself. Both de Broglie and Bohm [17, 18, 19], as well as another major supporter of this interpretation, Bell¹, realized this, and kept both the point particle and the wavefunction as ontic.

There are other reasons to take the wavefunction as ontic, rather than as a purely epistemic or probabilistic device, as implied by some results and no-go theorems [90, 55, 29, 30, 79, 54, 81, 74].

For all these reasons, I suggest that if we insist on assigning an ontological interpretation to quantum particles, this should be done by considering the wavefunctions or quantum fields as fields, similar to the classical ones. The wavefunction is normally defined on the configuration space and not simply on the physical space, but maybe this difference is not necessarily a problem. In fact, it is possible to faithfully represent the wavefunction in terms of fields on the physical 3D-space, and they are local under the unitary evolution [104]. Assigning an ontological interpretation to the wavefunction is not straightforward, and comes with some puzzles which I will discuss in section §4.

3. WHAT WOULD HAPPEN IF THE WAVEFUNCTION COLLAPSES WOULD BE DISCONTINUOUS?

Regardless of whether we take the wavefunction as ontic or not, it would be strange to have a unitary evolution law valid all the time except during measurements, when it is suddenly replaced, apparently without a cause, by a nonunitary projection or collapse. We expect the physical laws to be universal. So maybe there is an underlying universal law which appears like the Schrödinger equation almost all times, except for some discrete moments, when it appears like a projection.

But there is another problem: as long as we assume that the wavefunction collapses, the conservation laws are violated. We could guess this already, because in Quantum Mechanics the conserved quantities are those whose operators commute with the Hamiltonian, but during a collapse the Hamiltonian evolution is replaced by a projection, which does

¹According to Bell, “No one can understand this theory until he is willing to think of [the wavefunction] as a real objective field rather than just a ‘probability amplitude’. Even though it propagates not in 3-space but in $3N$ -space.” [15] p. 128.

not commute with the operators corresponding to conserved quantities. Such violations are shown to accompany the collapse explicitly in [103]. Moreover, it is shown that if we impose the spin conservation in the case of the spin measurements of spin 1/2 particles, there should be no collapse of the spin degrees of freedom of the wavefunction.

This violation of conservation laws due to a discontinuous collapse is true for the measurement scheme in the standard interpretation of Quantum Mechanics [103], but also for interpretations which take the wavefunction and its collapse as real, like the GRW interpretation [51, 50]. But, recalling that even in the de Broglie-Bohm interpretation, we have to assign physical properties to the pilot wave rather than to the point-particle, they are affected too (unless we reject their reality [34, 52], which does not solve the problem of the conservation laws). It is sometimes claimed that in the Bohmian interpretation there is no collapse, the only thing that happens instead being that the measurement makes all of the branches of the wavefunction except one simply become “empty”. Note that this emptiness is different from the update of information that the point-particle is in one of the branches and not the other, because if it was only about this information, then the empty branches would have been already empty before the measurement. But since before the measurement they have physical effects and interfere, and after the measurement they cease to have any effect, it is like they were there and then they disappeared. Once the measurement is done, the empty branches are effectively collapsed by plugging into the guiding equations the resulting position of the observed particle. And once a branch of the wave is emptied, the mentioned physical properties are no longer attributed to that branch, but only to the one in which the Bohmian point-particle was detected. So Bohmian mechanics simply cannot avoid the wavefunction collapse and the problems resulting from this, including the violation of conservation laws [103]. This also affects the Many Worlds Interpretation (MWI) [47, 48]. While MWI is unitary and without collapse as long as all branches or worlds are taken into account, collapse is still present within each of the branches. The fact that there is collapse within each branch implies that the result from [103] applies to each branch, hence conservation laws should be violated. Conservation laws are restored in MWI (and possibly in Bohmian mechanics) when all of the branches are taken into account, because unitary evolution remains valid.

It follows that the only way to avoid the violation of conservation laws would be if the time evolution would really be unitary, and the collapse never be real, but only appear as a collapse. It is indeed possible to have unitary evolution without collapse at the level of a single world, as explained in a series of papers by Stoica [93, 100, 94, 101, 102, 103]. Note that previously Schulman made a proposal based on special states which, as initial and final conditions, are required to be separable, and evolve unitarily (see [86, 87, 88, 89] and references therein). This is not the position taken here and in Stoica’s previous works, and the differences of motivation and implementation of the two approaches are discussed in [103]. In particular, the main difference consists of using global consistency between local solutions allowed by quantum measurements spread in spacetime, while Schulman’s approach is based on the initial and final conditions of separability of states.

Another problem appears when we take into account General Relativity. In this case, when the wavefunction collapses discontinuously, the stress-energy tensor operator $\hat{T}_{ab}$, and the expectation value $\langle \hat{T}_{ab} \rangle_{|0\rangle}$ of this operator collapses as well. This leads, via the

Einstein equation

$$(2) \quad R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi G\langle\hat{T}_{ab}\rangle_{|0\rangle},$$

to a discontinuous change in the Ricci tensor R_{ab} , hence in the spacetime curvature. This means a discontinuity of the covariant derivative ∇ . But the covariant derivative is involved in the time evolution equations of all particles. This means that the wavefunction collapse of a single particle should affect the time evolution of all other particles whose wavefunctions propagate in the region where the collapse happened. If this effect would be testable, it could be used to send superluminal signals. So probably it cannot be tested, or no collapse actually happens.

In addition, in [46] it is shown that QFT on curved spacetime where equation (2) holds could be used to send signals faster than c in a different way, and that it leads to violations of the uncertainty principle, assuming the wavefunction collapses discontinuously.

All these arguments lead to the question whether it is really necessary that the wavefunction collapses in a discontinuous way, or whether it is possible to avoid this and have unitary evolution even during quantum measurements.

4. WHAT HAPPENS IF THE WAVEFUNCTION COLLAPSE IS UNITARY?

The reasons mentioned so far justify us to at least consider seriously the possibility that the time evolution is always unitary.

The unitary time evolution of quantum systems is not an additional assumption, it follows from the Schrödinger equation and its relativistic versions. What I do is not to add a new assumption, but to argue that the assumption that unitary evolution is suspended during measurements and replaced by a discontinuous collapse of the wavefunction is not actually proven by experiments, and it was accepted too quickly. The idea that the state vector projection needs to be discontinuous is in fact a new assumption, a radical one, never proved directly, and, I argue, unnecessary. If we can show that the discontinuous collapse is unnecessary, new possibilities open up, including the possibility to combine Quantum Theory with General Relativity without sacrificing any of them.

But unitarity is a strong constraint, and its consequences have to be understood. Consider for example a measurement of the spin of a single particle. If the evolution is always unitary, and no discontinuous collapse happens, it means that the observed particle was already in a state which evolved in the observed eigenstate corresponding to the measured spin. We can account for the interaction between the observed particle and the measurement device, and such an interaction exists indeed, and changes the state of the observed particle. For example, the Stern-Gerlach device measures the spin of neutral atoms having a non-null magnetic moment by using the interaction between the particle and the magnetic field, so that the particle's spin changes. But the change is too small to bring the particle in an eigenstate of the spin along the chosen axis, if the previous spin of the particle was not already in an appropriate state which could evolve into the observed one. If the interaction taking place during the measurement is too large, then the Born rule is violated, in the sense that the spin can even be reversed from $|\uparrow\rangle$ to $|\downarrow\rangle$. Not any interaction counts as a quantum measurement.

The situation gets even trickier if we want to perform multiple non-commuting spin measurements on the same particle. The only way to do this without collapsing the spin in a discontinuous way is that the magnetic interactions with the two Stern-Gerlach

devices are fine-tuned so that the particle leaves the first device with deviated spin, and is then deviated more by the interaction with the second device, so that the total deviation changes the spin from an eigenstate of the first spin operator to an eigenstate of the second one. This fine-tuning of course requires that the particles in the two devices have fine-tuned states, as if they would conspire to give us the right outcomes.

If there is only unitary evolution, with no nonunitary collapse, then the total quantum state, containing the observed particle and the measurement device, should be in very special states, to allow for definite outcomes [100]. In fact, the measure of the initial states which can result in definite outcomes of the measurements rather than Schrödinger’s cat type of superpositions is zero compared to the measure of the entire Hilbert space, or at least almost zero, considering that many measurements are not exactly sharp due to limitations coming from the *Wigner-Araki-Yanase theorem* [122, 25, 5]. In other words, the initial state of the observed particle has to be perfectly synchronized with that of the measurement device, even though they are separated initially by a spacelike interval. There is no way to escape the apparent conspiracy between the observed particle and the measurement device, even if they come from causally separated regions of spacetime.

Such fine-tuning of the initial state is usually interpreted in terms of retrocausality. This may look very strange, but retrocausal models were already suggested by de Beauregard [35], and after that by Rietdijk [80]. Another approach, which provides a mechanism taking place in “pseudotime” is Cramer’s *Transactional Interpretation* [31, 32, 67]. Different models and approaches are proposed and discussed in [49, 118, 77, 106, 6, 78, 107, 1, 28, 119]. An interesting and important proposal, based on evolution in both directions of time, is the *two-state vector formalism* [2, 4, 27].

A way to understand retrocausality is by appealing to Wheeler’s delayed choice experiment [120]. In Wheeler’s experiment, the setup is such that we can choose between making a which-path measurement or an interference measurement, after the moment when the observed photon either took both ways or randomly only one. So it looks like the photon’s “choice” between the both-ways and the which-path possibilities is affected retrocausally by our choice of what experiment to perform. Of course, this experiment can be interpreted in terms of wavefunction collapse, but it is very eloquent in suggesting that a retrocausal effect takes place, which seems in this case a more natural explanation.

Later I will give an account of this apparent retrocausality, based on the block universe, which makes it less weird. But for the moment, let us face some more implications of this strange possibility.

An important feature of retrocausal approaches, including the purely unitary ones, is that they can recover both relativistic invariance and locality (or, we can interpret them as recovering locality in space, at the expense of “locality in time”). They are consistent with relativistic locality, and what they violate is *statistical independence*. As we remember from Bell’s theorem, there are two conditions leading to Bell’s inequality [14, 15]. The first condition is that of locality, and the second one is that of statistical independence. Statistical independence means that the initial state of the observed pair of particles is independent from the experimental setup. From these conditions, Bell’s inequality follows. But since the experimental evidence [8, 7] showed repeatedly that Bell’s inequality is violated, it means that at least one of the conditions of the theorem is violated, but we cannot say for sure which one. Retrocausal approaches violate statistical independence, and save locality. There is another implicit assumption in Bell’s theorem,

that the outcomes are determinate. If we assume that all outcomes happen, we obtain the Many Worlds Interpretation. I will come back to this later, because it provides a nice way out without having to choose between locality and statistical independence, and it also brings us a step closer to the unitary approach presented in this article. For the moment, we need to say more about retrocausality and locality.

Most physicists and philosophers of physics find it more acceptable to give up locality, and to maintain statistical independence. The reasons are obvious, violations of statistical independence seem to violate causality.

But a closer look considering relativistic causality in Minkowski spacetime shows that both options have similar problems. Nonlocality, coined by Einstein as “spooky action at a distance”, is at odds with relativistic causality. But this nonlocality does not allow us to send signals or energy outside the light cone by quantum measurements. This is considered to be consistent with relativity, of course, at an operational or epistemic level, not at the level of ontology. At the epistemic or operational level there is a “peaceful coexistence” between QM and SR, but if we want to avoid being instrumentalists about QM, we should not do so by becoming instead instrumentalists about SR, so I would argue that no-signaling is not enough, locality should be obtained at the ontological level.

But the same can be said about retrocausality. It does not violate causality, because it cannot be used to send information or energy back in time. It cannot be used to change the outcomes of already performed measurements. The experiments of quantum time travel [71] can be done, but there is no such observable violation of causality, just like in the quantum teleportation experiment no such observed violation of Einstein causality occurs [16, 110, 22]. But if we take seriously Einstein’s causality on Minkowski spacetime, and especially on curved spacetime, the option of rejecting locality is arguably more problematic than rejecting statistical independence. Both Special and General Relativity have well-defined ontologies, which are local. On a curved background, the fields whose stress-energy tensor corresponds to a spacetime curvature via Einstein’s equation should have well-defined local ontologies too. So it is irrelevant from this point of view that we can claim that violations of causality happen at the ontological level, but they are not observable. If we seriously adhere to the goal of providing an ontology for Quantum Mechanics, we have to take the ontology seriously all the time, and not conveniently ignore this ontology on the grounds that no quantum measurement can show that it violates Einstein’s causality. But the alternative way, of an ontology based on violations of statistical independence which does not violate locality, is not at all inconsistent at the ontological level with Einstein’s causality, especially in the block universe view. Retrocausal models have an advantage – ontologically, they are perfectly consistent with SR and GR, there is no need to invoke instrumentalist arguments to recover this consistency. The only reason to invoke no-signaling backwards in time is to recover causality, but this kind of causality is not necessary for SR or GR at a fundamental level, it is only needed at the coarse grained level. Indeed, both SR and GR are perfectly time symmetric theories, and they cannot distinguish past from future lightcones, so the covariant equations can be solved in both directions of time, and they allow influences in both directions of time. Not only are they consistent with bicausal interpretations, but they even endorse them. A direction of time is preferred because of the Second Law of Thermodynamics, but this preference is only statistical, and for the other laws retrocausal influences can always be reinterpreted as causal influences with special initial conditions.

Of course, to see that the present proposal indeed avoids the problems of nonlocality, we have first to make sure that the unitary evolution ontology is local. As I explained, the wavefunction is defined on the configuration space, rather than on the physical space. This means that the wavefunction is *holistic*. But even so, it admits a representation in terms of fields on spacetime, which propagate and interact locally, as long as the evolution is unitary [104]. Being holistic means that it allows entanglement, but since there is no wavefunction collapse, this entanglement is never projected to separable states. If this would have happened, then of course violations of locality would occur, because such a projection would have to be nonlocal. Even in the case of the EPR-B experiment this can be true, because it is possible to explain them as the particles not being actually in entanglement, but rather as if the singlet state decayed into two separate states, as if it already collapsed before the two particles went in separate places [35, 80, 78]. This interpretation seems to be confirmed through weak measurements [3] in the exact way that Bohmian trajectories are considered to be confirmed [69]. Hence, we can consider the processes taking place during the EPR-B experiment as being local in the sense that the particles are described by local solutions of the Schrödinger equation. This kind of spacetime locality is not what we usually expect when we speak about locality, because it depends on the final conditions imposed by the experimental setup. The solutions are local in the sense that they obey partial differential equations on spacetime, but they are also subject to boundary conditions which are global and impose the apparent (spatial) nonlocality like that from Bell’s theorem.

To understand how unitary evolution is local, let us first consider a wavefunction which is a separable state in a basis of eigenstates of an operator which commutes with the Hamiltonian. Since the relativistic evolution equation of such a state does not contain nonlocal interactions, this means that the state will evolve in a local manner, even though it is a multiparticle state. A general state is a superposition of such states, and each term of the superposition evolves locally. And since we cannot project any of them out, because we assume that all evolution is unitary and no discontinuous collapse occurs, this means that locality is ensured. Regardless of what other meanings we assign to locality, this type of locality is consistent with Einstein’s locality and causality. More about how the wavefunction can be represented in a locally separable way on the three-dimensional space, and how its unitary evolution is local, can be found in full mathematical detail in [104].

Since I mentioned Einstein’s locality and causality, it would be interesting which of the conditions from the 1935 EPR paper [41] is violated by this proposal. It is not difficult to see that Einstein’s criterion of reality is violated, but not in a “lethal” way. Indeed, our ontology does not satisfy Einstein’s realism, because the state before the measurement cannot be just any state, it depends on the experimental setup. This is another way to say that it violates statistical independence. Simply put, you cannot have any initial state for the observed particle and at the same time any initial quantum (microscopic) state of the apparatus. But this is not “lethal”, since it is perfectly consistent with relativity and Einstein’s causality and locality. In addition, as argued in [103], the macroscopic state of the apparatus and the quantum state of the observed system can be statistically independent.

Moreover, the avoidance of a discontinuous collapse also avoids the problems identified in Section §3, while a nonlocal ontology, at least in the currently known forms, cannot

do this, because there is no way to take the wavefunction as becoming empty of physical properties, and at the same time to say that this is not the same as collapse, as explained in Section §2.

But, even if we still find retrocausality preposterous, as I already alluded, there is a way to tame it, which is based on the fact that there is another hidden assumption in Bell's theorem, which requires that the outcomes are determinate. The violation of this assumption makes possible the Many Worlds Interpretation, which allows all outcomes to be obtained. Alternatively, one can also interpret MWI at the branch level, where the outcomes are determinate, as violating one of the two main assumptions of Bell's theorem. At first sight, it may seem that branching happens through some nonlocal collapse accompanying the measurement, and then in each branch locality is violated. However, the other option makes more sense. Consider the EPR-B experiment, and start from the outcomes obtained by Alice and Bob. When the two particles are detected, they no longer form an entangled state, and they no longer interact. Therefore, if we evolve the solution unitarily backwards in time, the two particles turn out to be separable and to not interact for the entire time interval between the decay that produced them and the measurement, so no collapse is required to occur except when the spin 0 particle decayed into two spin 1/2 particles. So it makes sense to consider that the collapse accompanying the branching took place, in a very localized manner, when the decay occurred, and the particles evolved unitarily until their detection. This makes some proponents say that MWI is local [9, 38, 111, 112]. A salient feature of this local interpretation of MWI is that, although in each branch retrocausality seems to happen, this does not really happen in the universal wavefunction, so overall, there is no violation of either statistical independence or locality, because all possible outcomes occur.

We have seen that it makes more sense that, in MWI, the split happens during decays and when systems are detected. While each branch taken independently manifests, by this, retrocausal features, on the overall there is no such thing. In the example with the EPR-B experiment, the branching seems to happen only during the decay into two spin 1/2 particles, and this can be understood by pushing unitary evolution back in time as much as it holds without contradicting the observations. But is it necessary to stop this at the decay? As it is understood from the Weisskopf-Wigner model of decay [117], while an isolated atom or composite particle should be stable, even if it is excited, vacuum fluctuations make it unstable, and this can be described as a superposition between the excited state and the decayed state (including the products of the decay). Then, the decay happens as a result of the amplitude damping of the excited state, so eventually the system decays. Observations of the system may introduce a collapse, which either makes the system decay, or makes it continue to remain in the excited state. But in the case of the singlet state used in the EPR-B experiment, there are different ways to decay, and the vacuum also is in an undetermined state. So it is possible that, when evolving backwards in time the two particles, one obtains a history in which the vacuum was in the right state required to provoke the decay of the singlet state exactly as needed for the subsequent measurements made by Alice and Bob to find the two particles in the right states. It is, at least in principle, possible that there is no actual branching at the fundamental level, but at the coarse-grained macroscopic level it would appear as if there is a collapse and that there is randomness. In this case, the MWI would be modified into a theory in which the worlds, rather than splitting, are really parallel and evolve unitarily,

but they are not always distinguishable at the coarse grained level. In addition, it is known that MWI has a problem with probabilities. While even since Everett's seminal work [47, 48] it was proposed to attribute probabilistic meaning to the amplitudes, this does not seem very appropriate, because the wavefunction is ontic. It would be perhaps more convincing to have a derivation of probabilities in terms of ensembles of microstates. But in a theory of truly parallel worlds, which only seem identical up to some point when measurements happen, and then they become distinguishable, probabilities would occur naturally in terms of ensembles of microstates. So, if we take the idea of parallel worlds which always evolve unitarily, we recover the conservation laws for each of the worlds, and also make room for probabilities. From all these arguments, it seems that if we push MWI to its consequences, each branch should in fact be independent and evolve unitarily for its entire existence. It remains an open problem how to derive the correct probabilities according to the Born rule in such a theory.

5. GLOBAL CONSISTENCY CONDITION

We have seen that, if the unitary evolution is not broken even during the measurements, then the initial states including both the observed system and the measurement apparatus (and the environment), have to be severely constrained, otherwise the outcome of the measurement will be undefined [100]. This looks like fine-tuning, superdeterminism, or retrocausality. We have seen in section §4 that MWI provides a nice way to tame retrocausality. But here I will discuss another possible explanation, which works for a single world, and makes sense in the block universe picture, and only looks like fine-tuning or retrocausality for the observers experiencing the flow of time.

Such an explanation is provided by *sheaf theory* [23]. Sheaf theory has many applications, but the one in which we are interested is how local solutions of partial differential equations (PDE) extend globally. When a PDE has well-defined initial conditions which ensure local solutions, this is not necessarily enough to ensure global solutions. Recall from Local or Algebraic Quantum Field Theory [53] that observables are local operators. Then, they can only give local information about the wavefunction. While this information is local, we need to extend the solution globally, and this has to be done consistently, so that we do not run into contradictions. For example, if a particle is found at a certain location, its amplitude at other locations should vanish. Similarly, in the EPR-B experiment, the local solutions found by Alice and Bob have to be mutually consistent, even if they are imposed at different locations. This is where sheaf theory becomes useful. Sheaves are collections of local solutions of PDE. When two local solutions have disjoint domains, or are equal on the common domain on which they are defined, they can be “glued” and extended to the union of their individual domains. But this extension of local solutions does not always work, because global extensions are not always guaranteed to exist. Even if the domains of the local solutions do not overlap, it is not always possible to have a global solution extending them. By this, sheaf theory shows us that, when the information we have about the wavefunction is spread in multiple locations, not all local conditions can be mutually consistent, so there are certain constraints, and correlations are enforced between local solutions. This is relevant for our discussion. For example, in the EPR experiment, we know that not all combinations of outcomes that Alice and Bob can obtain are consistent, and those that are consistent have different probabilities to occur. Global extensions of sheaves have similar properties. *Sheaf cohomology* studies

the obstructions which prevent the extension of local solutions to global ones. These obstructions are usually of topological nature. The space of initial or boundary conditions which admit global extensions is reduced, in the presence of such obstructions, to lower dimensional spaces. A simple example is that of *complex holomorphic functions*. They are complex functions satisfying the *Cauchy-Riemann* condition, which is equivalent to saying that such functions depend on $z \in \mathbb{C}$ but not on its conjugate. On the complex plane $\mathbb{C}$ they are spanned by the powers $n \geq 0$ of $z \in \mathbb{C}$, but the solutions do not always extend globally through analytic extension, they can run into singularities. Those with global extensions on $\mathbb{C}$ without singularities are called holomorphic, and their space is smaller than the space of functions spanned by the powers of z , yet still infinite-dimensional (the polynomials in z form an infinite-dimensional subspace of the space of holomorphic functions on $\mathbb{C}$). But on the Riemann sphere $\mathbb{C} \cup \infty$, the only holomorphic functions are the constant ones, so their space is one-dimensional. Changing the topology by adding a single point introduces incredible constraints. Such examples suggest an interesting possibility: what if there are constraints, perhaps topological in nature, at the level of the spin and gauge bundles and especially interaction bundles used to describe particles and forces, which ensure that only certain solutions are possible? Could the global solutions correspond to the definite outcomes of measurements, and explain the correlations between outcomes obtained when measuring entangled systems? From the mathematical examples we have in sheaf theory, this appears to be a plausible possibility, and it makes sense to explore it as a candidate solution of the measurement problem and the problem of the apparent classicality at the macroscopic level. But it is difficult to know for sure, until we will have a better understanding of the fiber bundle structures, and of what kind of constraints they impose to the global existence of solutions.

It is interesting that Schrödinger used a consistency condition on the space to obtain the energy eigenstates of the Hydrogen atom, in the form of a boundary condition of the wavefunction of the electron at infinity [85]. The usual view on this is in terms of eigenstates of the Hamiltonian, but in terms of a wavefunction on space, this turned out to come from boundary conditions at infinity. The requirement of global consistency is similar, but on spacetime rather than space, in fact, on the spinor and gauge bundles.

One may think that the example of complex holomorphic functions is irrelevant to Quantum Mechanics for two reasons. First, what is the relevance of the Cauchy-Riemann condition? Well, the Cauchy-Riemann operator is for two real dimensions what the Dirac operator is for Minkowski spacetime, as it is known from the theory of Clifford algebras [26, 33]. Moreover, the same operator can be used to express the Maxwell equations in a more compact way as a single equation [64], and this also works for the Yang-Mills equation.

A second objection could be that Minkowski spacetime has a trivial topology, and even the curved spacetime probably has, if not a trivial topology, maybe a simple one. And this is true, but on the other hand in reality both the existence of spin 1/2 particles, and of the internal gauge degrees of freedom, require the existence of fiber bundles, and they have a complicated topology. Unfortunately, it is not currently understood exactly what happens from a topological point of view, and consequently we do not know the conditions to be satisfied for a solution to be global. But the point is that the Hilbert space is severely reduced, and we need to see exactly how. Future understanding of the geometric and topological properties of particles and their interactions, especially in a

quantized theory, may shed light on this issue and explain both why the quantum states appear classical at a macroscopic level, and how quantum measurements yield definite outcomes without invoking the wavefunction collapse.

But what is relevant to our discussion is that, if this is the case, then we have an explanation for the restrictions on the initial conditions: only those initial conditions leading to globally consistent solutions on the entire spacetime are admitted. And while such severe restrictions appear as a conspiracy to an observer experiencing the flow of time, a bird's eye view of the block universe would see everything just as the natural condition that the solutions are global.

Hence, despite the fact that the block universe is sometimes seen as being at odds with Quantum Mechanics, it complements it and offers a possible solution to its problems.

6. QUANTUM PROBABILITIES

The solutions of Schrödinger's equation are unitary, but when we think about “wavefunction”, we think at two different things. On the one hand, as long as no measurement is made on a quantum system, we can regard the wavefunctions as a function on the configuration space, but it also admits a representation as fields on spacetime [104]. On the other hand, no measurement can completely determine the quantum state of the entire system made of the observed system and the measurement apparatus (and other relevant parts of the environment). This means that our measurements cannot be used to determine the future outcomes of the measurements of the observed system, but only probabilities. I do not yet have a proof whether these probabilities are exactly those given by the Born rule or not, but the possibility to recover the Born rule exists. The real, “ontic” wavefunction will be never completely determined, but what we can know is an “epistemic” wavefunction. The notions “ontic” and “epistemic” may be used differently by different authors, but I will stick with the definition that there is a real, “ontic”, physical wavefunction, and “epistemic” is the partial knowledge of the ontic wavefunction of the universe, which translates as probabilities when it comes to learn more about it through measurements.

Some usual terms associated to the wavefunction have a statistical connotation: expectation value, uncertainty, etc. These terms retain their statistical meaning when we are talking about the epistemic wavefunction, which is probabilistic. But when we are talking about the ontic wavefunction, the “expectation value” of an operator $\hat{O}$ simply means the field $\langle\psi|\hat{O}|\psi\rangle$, and similarly for uncertainty and other terms.

In particular,

$$(3) \qquad \langle\hat{T}_{ab}\rangle_{|0\rangle} = \langle 0|\hat{T}_{ab}|0\rangle$$

is not a true expectation value, but, after suitable regularization, a physical field on spacetime. This will be relevant in the next section.

7. QUANTUM THEORY AND GENERAL RELATIVITY

Despite the measurement problem and the problem of the emergence of the classical world of Quantum Mechanics, Quantum Field Theory is incredibly successful in describing the microscopic scale. At large scales, General Relativity does the job with equal success. Both theories were extensively tested, with great precision and in diverse situations, and their predictions turned out to be accurate every time. Not everything is understood, for example in cosmology there are the problems of inflation, dark energy, and dark matter.

We do not now yet if they require changing the Standard Model, Quantum Theory, or General Relativity.

However, since both theories have to be true, we need to understand how they work together. When we try to combine them, they seem to be in a conflict. The general trend is to consider that one of the two theories will have to be radically changed, or even replaced, and that this one is GR. The main invoked reasons are the successes of Quantum Theory, the prediction of singularities in GR, the black hole entropy, and the information loss paradox. But mainly the resistance of gravity to be quantized in the same way as the other forces.

The predictions of QFT are confirmed to great accuracy, in particular in the case of the anomalous magnetic dipole moment. On the other hand, GR's predictions are confirmed with at least the same accuracy, and even better in the case of the Hulse-Taylor binary pulsar PSR 1913+16 [62]. I think both theories were confirmed incredibly well in all predictions that could be tested. So it would not be fair to hold the success of QFT against GR.

It is true that GR predicts the occurrence of singularities [76, 56, 57, 58, 61]. But QFT is plagued with infinities too, in both the UV and IR regimes. It is true that renormalization worked particularly well in the predictions of great precision, and the *renormalization group* (actually it is a semigroup) idea provides a deeper understanding, but it is not as if the infinities are completely cured. In fact, they are rather an artifact of the perturbative expansion. But the singularities in GR are not worse at all. Indeed, it is more difficult to see this, but it turns out that differential geometry [98], and in particular GR [96], can be formulated in a completely invariant way which is equivalent to the standard formulation outside singularities, but which allows us to write field equations, including an equation equivalent to Einstein's, even at singularities (see [95] and references therein). Moreover, the same solution of the problem of singularities was used as an ingredient in an explanation of the observed values of the expansion rate of the universe [116, 115].

As for the problem of the quantization of gravity, do we really need to do in the same way as other forces are quantized? They are of apparently different nature, gravity is due to spacetime curvature, and the other forces are gauge fields. There seems to be no *a priori* reason to do the same for gravity, which is just inertia on curved spacetime. Could the theory of quantum fields on curved background, where the expectation value of the stress-energy operator is introduced in the Einstein equation (2), be enough? This is usually called *semi-classical gravity* [70, 82], and is considered to be an approximation of the true quantum gravity which remains unknown. But could it be enough?

In the case of a single particle which never collapses, it seems at first sight that there is no problem with semi-classical gravity: even if the particle is set in various superpositions, what matters for GR is its stress-energy tensor. If there are more particles, or even an undefined number as it is often the case in QFT, particularly on curved spacetime, the expectation value of the stress-energy operator does the same job. And since there is never a wavefunction collapse even though the macroscopic world seems classical and the measurements have definite outcomes, the stress-energy tensor is conserved and behaves as any classical source for gravity and spacetime curvature.

There are several arguments usually invoked that semi-classical gravity is not enough.

In [46], it is argued that if gravity is classical, one could use it to measure a quantum state in a way which violates the uncertainty principle. Two cases are identified. (1) If

the wavefunction of the observed system is collapsed during the classical measurement with a gravitational wave, such a measurement could lead to violations of the momentum conservation. (2) If the classical measurement does not collapse the wavefunction of the observed system, then we could use it to send superluminal signals. To prove this argument, one considers a particle in two boxes, one given to Alice, and another one to Bob. Then, Bob can “look” inside his box using arbitrarily non-disturbing classical measurements, and determine the state of the wavefunction in his box without collapsing it. By this, he would be able to know if Alice looked in her box. So, it is concluded, semi-classical gravity may lead to the transmission of signals faster than c .

But if there is no wavefunction collapse, option (1) is easily avoided without quantizing gravity. At first sight it may seem that option (2) is avoided as well, because even if Bob does not collapse the wavefunction, Alice does it.

However, in our approach, measurements can introduce challenges. We have seen that if quantum measurements take place by unitary evolution, the present state of the particle should depend on the future measurement. So, if we can use such weak classical gravitational fields to determine the state of a particle without disturbing it too much, we can extract information about the choice of the future quantum measurement and its outcome. Since this choice is supposed to be free, it can be used to send information back in time. It can be argued that this situation is similar to the weak measurements [108] of the state before undergoing a quantum measurement – both theory and experiments show that there is a correlation between the weak measurements and the future quantum measurements, but it is not enough to signal back in time [3]. However, if by using classical fields one can extract more information about the state while maintaining the disturbance small enough, one could be able to signal back in time. Therefore, at least for this case, the most natural way out of this problem seems to be to quantize somehow the gravitational field, so that any measurement we make using gravity disturbs the observed system like quantum measurements do.

In [75] an experimental result is described, testing whether semi-classical gravity with no wavefunction collapse holds (although the authors assume that the unitarity could only be ensured by the Many Worlds Interpretation). The result is interpreted as a falsification of semi-classical gravity because no superposition of macroscopic massive objects at different locations was found. However, there is an alternative explanation favorable to semi-classical gravity without collapse, if we assume unitary evolution for a single world, such that macroscopic massive objects do not end up in a superposition of being in different positions. According to the proposal supported in this article, global consistency may prevent Schrödinger cats, including superpositions of macroscopic massive objects at different locations. Hence, if the hypothesis that global consistency allows for a unitary solution which never collapses discontinuously, and at the same time prevents macroscopic superposition, is true, no serious theoretical or experimental evidence against semi-classical gravity is found in [46] and [75].

Therefore, the absence of a collapse makes Quantum Theory and GR more compatible. But this does not exclude the possibility that quantization of gravity is still needed, and in fact still seem to require it.

It is known that, normally, quantization of gravity cannot be done perturbatively. However, various suggestions of modifications are made, going under the common umbrella of *dimensional reduction* techniques. They are based on *ad-hoc* assumptions which have

the purpose to reduce the divergences as the perturbative expansion goes to the UV limit [97]. It turns out that there is no need to make these *ad-hoc* hypotheses, since the treatment of singularities given in [95] ensures automatically the conditions imposed in several of these approaches, assuming that, in the UV limit and in a position basis, particles behave more and more like tiny black holes with singularities [97].

Another often-encountered argument is that GR has to be changed, because if we would try to experimentally probe spacetime at Planck scales, we would not be able to do it, because the high energies involved would lead to the creation of micro black holes. While indeed this will be the case, this only sets a limitation on our experimental possibilities, one not due to our limited technology, but to principial limitations due to backreaction. But the universe is under no contractual obligation to allow us to probe spacetime in these regimes. Such a limitation of our experimental possibilities does not require replacing spacetime with something else, especially since for anything else we would have the same experimental limitations.

Another reason sometimes invoked when semi-classical gravity is said to be only an approximation comes from the *black hole entropy*. It is said that the event horizon is homogeneous, and it would not be able to store information, so either the horizon or the spacetime inside the black hole have to be discrete to represent microstates which would give the right entropy. But the entropy calculations are done with the help of QFT on a spacetime regime. The black hole entropy is calculated based on the quantum information of the particles falling in the black hole [12, 10, 114, 66]. Hence, the black hole entropy was derived in the strict framework of QFT on a curved background. The same holds for *black hole evaporation* – it was derived in the same framework [59, 60]. This means that both black hole entropy and black hole evaporation are predicted and explained by QFT on spacetime already. Then why would we need another theory to derive the same predictions [105]? Moreover, the information is considered lost only because the black hole singularities seem to lead to Cauchy horizons, but if the global hyperbolicity is not necessarily destroyed by singularities, as explained in [95], then this problem can be solved within the framework of GR itself [99, 105].

All these arguments concur in showing that the main problems we tend to think to require a Quantum Theory of spacetime itself may in fact be solved within QFT on curved spacetime. There is no necessary reason to think otherwise. Surely, it may still be true that there is a better theory, maybe one of the proposals of quantum gravity or maybe one we do not know yet, but the currently known arguments no longer seem to be so strong.

8. THE POST-DETERMINED BLOCK UNIVERSE

As humans, very early in life we become aware that events that already happened cannot be changed, and that future events, although unpredictable, can be influenced by our present actions. This intuition is so deeply hardwired in our world view, that it seems unnatural to even question the idea that past and future do not exist, but only present does. Therefore, it comes as a surprise that thinkers like Parmenides, Boethius, Augustine of Hippo, Anselm of Canterbury, Dōgen, and others either claimed, or seem to suggest the opposite view, that past, present, and future are equally real. The first, more common position, is called *presentism*, while the second position is called *eternalism*. A major advocate of eternalism was McTaggart [72].

With the discovery of classical mechanics, and its determinism, we started to realize that, at least mathematically, the past, present, and future states of the universe are encoded in the state of the universe at any time t_0 . As Laplace explained it, if the state of the universe as a physical system is known with perfect accuracy at a time t_0 , and the equations or motions are known, one can, in principle, calculate the state of the universe at any other time. This is a property of the differential equations expressing the physical laws of classical mechanics. What appears to us as random events are in fact due to the absence of complete information about the state. So it is surprising that, knowing that past and future are encoded in the present, physicists did not consider eternalism as a viable option. We had to await the discovery of Special Relativity, with its relativity of simultaneity, to take this idea seriously. Accordingly, if two observers in relative motion agree that a certain event took place at a certain time, they will in general disagree about what other events happened at the same time with the first one. Spacetime as a block universe (BU) appeared gradually in the works of Lorentz, Poincaré, Einstein, and Minkowski [73]. The idea of a spacetime in which time and space are inseparable seemed too farfetched to many physicists, who either rejected Special Relativity, or considered spacetime as a convenient mathematical tool, but eventually the BU and the eternalist position started to be seen as endorsed by Special Relativity.

One of the major objections against the BU was its apparent incompatibility with *free-will* (a notion which I do not know how to define or explain). A hybrid proposal was made by C.D. Broad in 1923, the *growing BU* [24]. According to this proposal, both past and present exist, but the future does not yet exist. The past is a block which grows continuously with the passage of time, which is connected to the human experience. In a totally different direction, to reconcile free-will with classical determinism, *compatibilists* took the position that freedom means acting according to one's nature, and thus determinism is not only consistent with freedom, but it allows it to be expressed. This compatibilist position is consistent, in particular, with the BU. But Hoefer took a step beyond compatibilism, and proposed that we are even free to make choices affecting the initial conditions of the universe [65], within the framework of the BU of classical relativity.

Soon after the discovery of Special Relativity, Quantum Mechanics appeared, with its probabilistic Born rule and Heisenberg's uncertainty principle. This time, it seemed difficult to explain the probabilities as a mere lack of knowledge, and they came to be understood as irreducible. Results like Bell's theorem and the Kochen-Specker theorem [68, 13] were taken by many as endorsing this position. This happened despite the fact that at least a deterministic interpretation of QM was known at that time, the pilot-wave theory of de Broglie and Bohm [17], which is consistent with both no-go theorems, being both nonlocal and contextual. The presentist position seemed to win. Because of this, it is usually believed that the BU picture can only hold for classical GR, but quantum indeterminism is necessarily incompatible with it. This is sometimes taken as evidence that the *eternalism* of GR does not hold, and it should be replaced by *presentism*, which can be expected to be consistent with the wavefunction collapse and with nonlocal interpretations of QM. But the evolving or growing BU model [24] was seen by Ellis as consistent with QM, since the growth can be seen as taking place in a nonlocal and indeterministic way as new quantum measurements are performed [42, 43]. Later, in 2009, Ellis and Rothman proposed a *crystallizing BU*, which captures the "quantum transition from indeterminacy to certainty" [45]. This model also includes retrocausal influences,

and the crystallization does not always occur simultaneously, in some cases having to wait for future experiments to be done, as it is the case with the delayed choice experiment [120]. Thus, the model provides support in particular to retrocausal interpretations of QM in which there is collapse, the collapse leading to the growth of the past block, and quantum indeterminacy allowing the future to be open.

It is possible to conceive a *splitting* or *branching BU*, which would be the BU version of the Many Worlds Interpretation, see for example [109], and for some criticism [44]. One can imagine a branching BU, in which each branch is an evolving or crystallizing BU, but on the overall they are all part of a tree-like structure. But there is a problem: in MWI, the wavefunction is real, but it is defined on the configuration space. Even for each branch, there is entanglement, at least because each branch contain atoms, which contain entangled particles. So, the wavefunction has to be replaced somehow, with an equivalent structure defined on space or spacetime, rather than on the configuration space. In fact, this applies to the other interpretations which make use of the wavefunction. Fortunately, as I already mentioned, there is a way to replace the wavefunction by a high-dimensional field defined on space, which is mathematically equivalent to the wavefunction on configuration space [104].

Despite the tremendous success of nonrelativistic Quantum Mechanics in describing microscopic physical phenomena, the domain of high energies requires something more. Quantum Field Theory appeared as a relativistic quantum theory. Wigner and Bargmann classified elementary particles in terms of representations of the Poincaré group [121, 11]. Thus, as long as there is no collapse, QFT is consistent with relativistic invariance, and it even requires it at the deepest level. But the wavefunction collapse would be in tension with this relativistic invariance, required by the very definition of particles in terms of representations of the Poincaré group. This is another reason to take unitary evolution seriously, and this brings us to the central point of this article.

In this article, I argued that in Quantum Mechanics it is possible for the time evolution to be unitary, without discontinuous collapse, at the level of a single world. I argued that this has some advantages, in particular it allows us to have a single law which is not broken even occasionally, it ensures the conservation laws, avoids nonlocality, and makes QFT more consistent with GR, but seems to suggest that the initial conditions have to be very specially chosen, to conspire to make measurements have definite outcomes rather than resulting in Schrödinger cats. But these apparent conspiracies seem rather natural if we see them as the consequences of global consistency in a BU. I hypothesized that the global consistency is about the consistency of solutions in the presence of topological constraints of the fiber bundles from spin geometry and gauge theory, and that it manifests itself by reducing the Hilbert space to a subset which does not contain Schrödinger cats and where all quantum measurements have definite outcomes.

These arguments suggest the following picture of a *post-determined block universe*. There is a BU, on which QFT is true and the average of the stress-energy operator connects to the spacetime curvature via the Einstein equation (2). Yet not all possible initial conditions can be true, but only those which lead to globally consistent solutions. So we start with a set of possible BUs consistent with the previous quantum observations, and as we make new observations, we refine that set. So far it looks quite like classical physics, but since the initial conditions are severely constrained by global consistency, the resulting correlations can be expected to violate the Bell inequality and its generalizations in a

way in which a dynamical system with no global constraints of the initial conditions could not be able to violate. This allows us to have a picture in which quantum measurements happen as predicted by the projection postulate, without breaking the unitary time evolution. In this picture, the BU is not pre-determined, but it is gradually post-determined as we make new quantum observations.

This kind of BU is deterministic, but it is not predetermined in the usual sense. The initial conditions are determined with a delay, by each new measurement and each choice of what to measure. The requirement of global consistency implies a severe restriction on the possible solutions of the Schrödinger equation, but since the observers can choose what to measure, it looks like they determine the past initial conditions more, with each new choice. The solution is still deterministic, but it is determined by future choices and the outcomes of measurements. We can still think of this proposal as including a form of superdeterminism or retrocausality, if we assume that the initial conditions are fixed from the beginning. But we can also take the position that the quasi-classical limit, which is a coarse graining of the low-level quantum state, evolves by usual causality in an indeterministic way. As observers, we start with the full set of quantum states consistent with our previous macroscopic observations, and then reduce them as new measurements provide more information. And since we never know the true quantum state, but only outcomes of our observations made on subsystems, these observations allow us to predict only probabilities, or an epistemic wavefunction which is an approximation of the ontic wavefunction, and has to be readjusted after each new measurement by invoking the wavefunction collapse.

For the reader concerned about the determinism inherent in the post-determined BU, there is freedom in the initial conditions. The proposal is consistent with “free-will” like in Hoefer’s model [65], but, in addition, it works in the context of QM, to explain measurements by unitary evolution alone. In relation to unitary evolution without collapse, this type of BU was suggested in various places by the author [93, 91, 92, 94, 101]. The post-determined BU can be compared with the crystallizing BU proposal [45], since the latter also has retrocausal influences due to delayed choices, but in the post-determined model, these apparent retrocausal influences apply to the entire past history of the universe. The post-determined BU can also be compared with the splitting BU of MWI, but there is no splitting at the fundamental level, only at the coarse grained level we can consider that there is branching. We start with a set of possible block universes consistent with our current observations, and as we add new observations, we refine the set of possible block universes, by eliminating those that are inconsistent with the new data. Indeterminism, which can also be interpreted as branching, is manifest only at the macroscopic or coarse grained level. In the post-determined BU, there is no actual growth or crystallization or branching at the fundamental level, but only at the coarse grained level accessible to the observers. The post-determined BU accommodates the main advantage of these proposals, which is their consistency with Quantum Mechanics, by solving the problems mentioned in §3, and restores the full advantages of the BU picture at the same time.

The post-determined BU is as deterministic and fixed as the standard one from the bird’s eye view of someone who knows completely the ontic wavefunction of the universe. From the point of view of someone who is part of the universe itself, like us, it may look as a growing BU, with the amendment that the growth is not only towards the future, but

at the quantum scale, because of global consistency, it also seems to be growing towards the past, giving the impression of retrocausality. But this retrocausality is not accessible to us to send messages into the past or at a distance, being forbidden by the fact that we only have “clearance” to approximate eigenstates, and not to the full quantum state of the observed systems.

By eliminating the discontinuous collapse, we remove important obstructions that seemed to put Quantum Theory and General Relativity at odds with each other. The so-called *semi-classical gravity* can now be more than an approximation of a future theory of quantum gravity, or at least it can be a better approximation than we used to think. With an ontic wavefunction, the “expectation value” of the stress-energy operator is not a probability, but a field, and we can plug it into Einstein’s equation and get a well-defined classical geometry. This does not imply that there is no need to quantize gravity, but rather that such a quantization has to solve fewer problems, because some of them are already avoided by avoiding the collapse and by allowing matter to be described by an ontic wavefunction.

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